

# On SkipGram Word Embedding Models with Negative Sampling: Unified Framework and Impact of Noise Distributions

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## Introduction

SkipGram word embedding models with negative sampling [1] (SGN) is an elegant family of word embedding models. In this work, we ask the following questions.

- Beyond that particular distribution, if one chooses a different noise distribution, is SGN still theoretically justified?
- Is there a general principle underlying SGN that allows us to build new embedding models?
- If so, how does the noise distribution impact the training of such models and their achievable performances?

## Our Contributions

- We formalize a unified framework, referred to as “word-context classification” (WCC), for SGN-like word embedding models.
- We also provide a theoretical analysis that justifies the WCC framework. Consequently, the matrix-factorization result of [2] can also be derived from this analysis as a special case.
- The impact of noise distribution on learning word embeddings in WCC is also studied.

## The WCC Framework

**Binary classification problem**  $D^+$  and  $D^-$ :

- **Objective:** distinguish the word-context pairs drawn from  $\mathbb{P}$  from those drawn from  $\mathbb{Q}$ ;
- The classification problem is equivalent to learning the conditional distribution  $p_{U|XY}(1|x, y) := \sigma(s(x, y))$

**WCC:** Let  $f: X \rightarrow \bar{X}$  and  $g: Y \rightarrow \bar{Y}$  be two functions representing the embedding maps for words and contexts respectively. The standard cross-entropy loss for this classification problem is

$$\ell = - \sum_{(x,y) \in D^+} \log \sigma(s(x, y)) - \sum_{(x,y) \in D^-} \log \sigma(-s(x, y))$$

and the solution is

$$(f^*, g^*) := \arg \min_{f, g} \ell(f, g)$$

**Theorem 1:** Suppose that  $\tilde{\mathbb{Q}}$  cover  $\tilde{\mathbb{P}}$ . Then the following holds.

1. The loss  $\ell$ , as a function of  $s$ , is convex in  $s$ .
2. If  $f$  and  $g$  are sufficiently expressive, then there is a unique configuration  $s^*$  of  $s$  that minimizes  $\ell(s)$ , and the global minimizer  $s^*$  of  $\ell(s)$  is given by

$$s^*(x, y) = \log \frac{\tilde{\mathbb{P}}(x, y)}{\tilde{\mathbb{Q}}(x, y)} + \log \frac{N^+}{N^-}$$

for every  $(x, y) \in X \times Y$ .

**Corollary 1:** Let  $N^+ = n$  and  $N^- = kn$ . Then it is possible to construct a distribution  $\tilde{\mathbb{P}}$  on  $X \times Y$  using  $f^*$ ,  $g^*$ ,  $k$  and  $\mathbb{Q}$  such that for every  $(x, y) \in X \times Y$ ,  $\tilde{\mathbb{P}}(x, y)$  converges to  $\mathbb{P}(x, y)$  in probability as  $n \rightarrow \infty$ .

## SGN Model

Let  $\mathbb{Q}$  factorize in the following form

$$\mathbb{Q}(x, y) = \tilde{\mathbb{P}}_X(x) \mathbb{Q}_Y(y)$$

**Corollary 2:** In an unconditional SGN model, the global minimizer of loss function  $\ell$  is given by

$$s^*(x, y) = \bar{x} \cdot \bar{y} = \log \frac{\tilde{\mathbb{P}}(x, y)}{\tilde{\mathbb{P}}_X(x) \tilde{\mathbb{Q}}_Y(y)} - \log k$$

As a special case when  $\tilde{\mathbb{Q}}_Y(y) = \tilde{\mathbb{P}}_Y(y) \Rightarrow$  “PMI”!

## Conditional SGN Model

Let  $\mathbb{Q}$  factorize in the following form

$$\mathbb{Q}(x, y) = \tilde{\mathbb{P}}_X(x) \mathbb{Q}_{Y|x}(y)$$

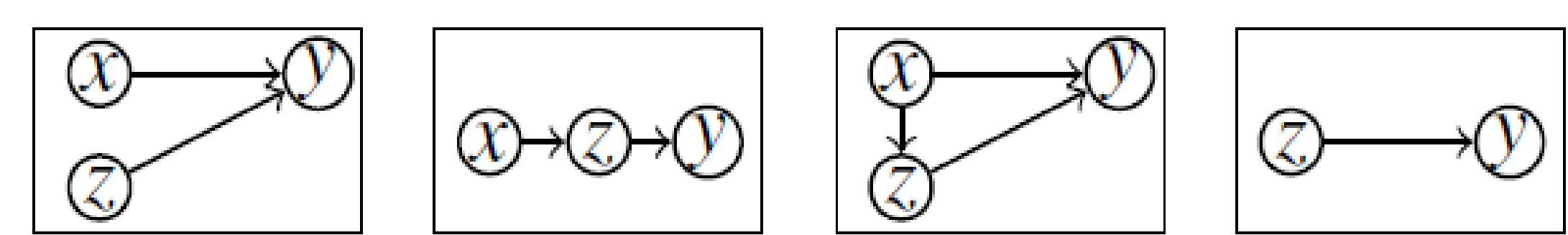
**Remark 1:** Consider some  $(x, y) \in \text{Supp}(\tilde{\mathbb{Q}}) \setminus \text{Supp}(\tilde{\mathbb{P}})$ , namely,  $(x, y)$  is “covered” by  $\tilde{\mathbb{Q}}$  but not by  $\tilde{\mathbb{P}}$ . Then the gradient is

$$\frac{\partial \ell}{\partial s(x, y)} = \sigma(s(x, y)) \cdot N^- \tilde{\mathbb{Q}}(x, y)$$

This may result in slow training.

**Hypothesis:** The best  $\tilde{\mathbb{Q}}$  is the one that barely covers  $\tilde{\mathbb{P}}$ , namely, equal to  $\tilde{\mathbb{P}}$ .

Under this hypothesis, choose  $\mathbb{Q}_{Y|x}$  to closely resemble  $\tilde{\mathbb{P}}_{Y|x} \Rightarrow$  **GANs** [3]!



(a) caSGN1 (b) caSGN2 (c) caSGN3 (d) aSGN

## Experiments

Table 1: Spearman’s  $\rho$  (\*100) on the word similarity tasks (text8).

Models	WS-353	WS-SIM	WS-REL	MTurk-287	MTurk-771	RW	MEN	MC	RG	SimLex
SGN	70.58	74.54	68.10	64.29	55.59	36.63	62.16	60.82	60.17	29.69
ACE	71.49	74.61	69.50	65.52	56.63	37.85	62.75	62.65	62.39	30.37
aSGN	71.12	74.76	68.82	65.07	56.47	37.58	62.63	62.36	62.36	30.49
caSGN1	71.72	75.11	69.77	65.63	56.63	37.63	63.40	62.54	64.18	30.36
caSGN2	72.02	75.05	69.64	65.44	57.02	37.61	63.36	62.86	64.63	30.79
caSGN3	71.74	74.61	69.63	65.57	56.56	37.78	62.69	62.61	62.52	30.31

Table 2: Spearman’s  $\rho$  (\*100) on the word similarity tasks (wiki).

Models	WS-353	WS-SIM	WS-REL	MTurk-287	MTurk-771	RW	MEN	MC	RG	SimLex
SGN	67.49	74.61	61.51	63.00	59.24	39.99	68.73	64.47	69.59	31.37
ACE	71.03	76.23	66.24	63.05	60.27	40.02	67.55	76.60	70.01	31.10
aSGN	70.66	75.69	65.69	65.14	61.22	39.81	68.99	77.38	73.67	31.58
caSGN1	71.56	76.05	65.37	63.05	61.63	40.74	69.72	80.07	77.58	31.27
caSGN2	70.57	73.84	65.83	65.26	62.28	41.24	70.93	71.57	73.05	30.45
caSGN3	70.27	74.93	65.26	65.98	59.52	41.55	70.05	75.95	73.52	31.40

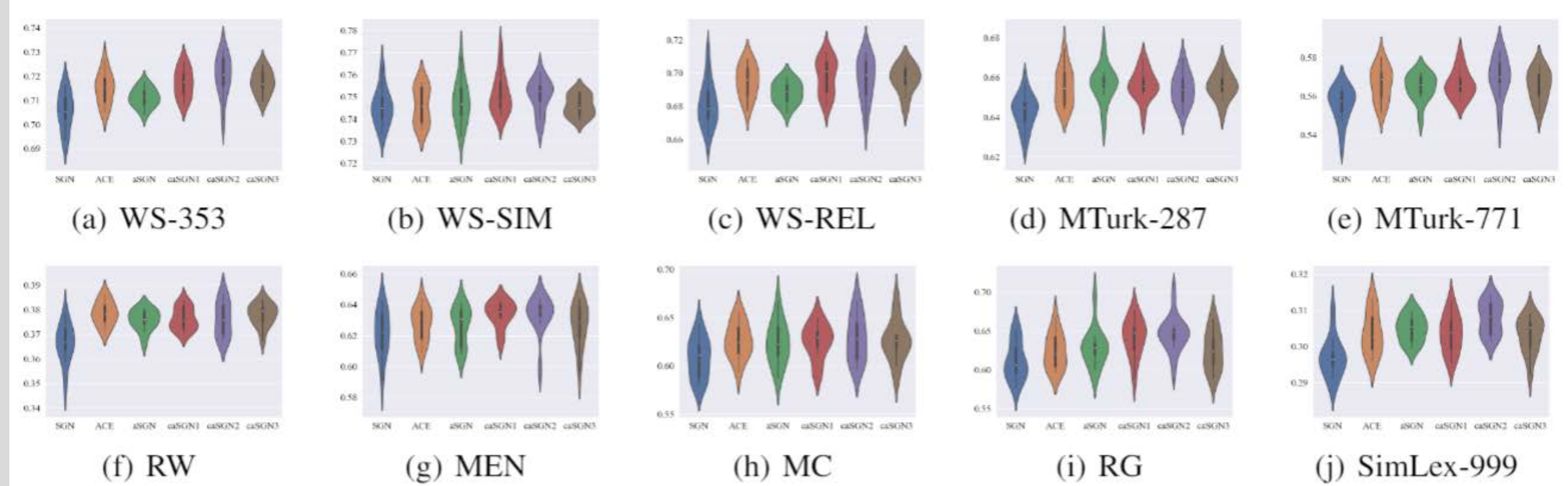
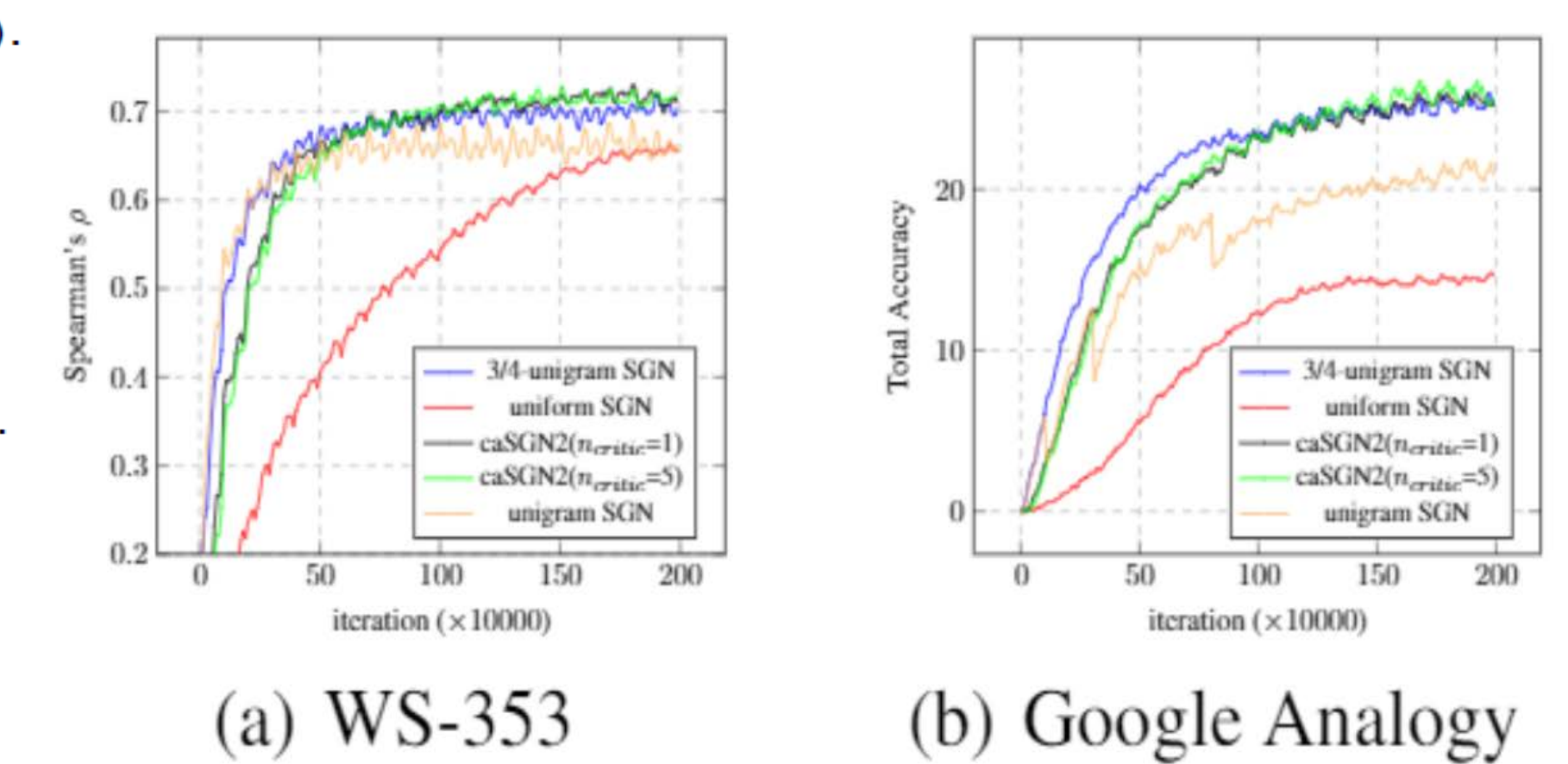


Table 3: Accuracy on the word analogy task (text8).

Model	Semantic	Syntactic	Total
SGN	20.50	26.77	24.16
ACE	20.43	28.25	25.00
aSGN	20.84	27.86	24.94
caSGN1	21.25	28.30	25.36
caSGN2	21.56	27.79	25.20
caSGN3	20.43	27.76	24.71

Table 4: Accuracy on the word analogy task (wiki).

Model	Semantic	Syntactic	Total
SGN	27.28	35.52	31.77
ACE	27.62	35.30	31.81
aSGN	35.24	38.66	37.10
caSGN1	31.71	38.32	35.31
caSGN2	37.00	39.96	38.61
caSGN3	41.21	39.24	40.14



## References

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